

Prof. Spera

FACOLTÀ DI SCIENZE MM.FF.NN.
Corso di laurea in Scienze Biologiche

Prova scritta di Matematica
del 9-06-2010

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1. Provare che il prodotto di due funzioni continue in $x_0 \in \mathbb{R}$ è continua in x_0 .

2. Studiare la seguente funzione:

$$f(x) = \frac{3x}{\sqrt{x^2 + 1}} - 2.$$

3. Calcolare la derivata seconda della seguente funzione:

$$f(x) = \frac{4}{3x - \sin x} + 2x. \quad \text{Solto}$$

Determinare inoltre (se esiste) la retta tangente al grafo di $f(x)$ nel punto di ascissa $x_0 = \pi$.

4. Calcolare il seguente integrale definito:

$$\int_0^{\frac{\pi}{2}} [e^{3x} \cos(4x) + 3x + 4] dx.$$

5. Calcolare il seguente limite:

$$\lim_{x \rightarrow -\infty} \left[\frac{3x - 4}{\sqrt{16x^2 + 3x + 2}} + 5 \right].$$

Inoltre determinare tutti gli asintoti della funzione $f(x) = \frac{3x-4}{\sqrt{16x^2+3x+2}} + 5$. solo orizzontali

$$p(x) = \frac{3x}{\sqrt{x^2+1}} - 2$$

$$\text{DOMINIO} \begin{cases} x^2+1 \geq 0 & \forall x \\ \sqrt{x^2+1} \neq 0 \Rightarrow x^2+1 \neq 0 & \forall x \end{cases} \Rightarrow]-\infty; +\infty[$$

T. ASSI

INT. ASSEX

$$\begin{cases} y=0 \\ y = \frac{3x}{\sqrt{x^2+1}} - 2 \end{cases} \Rightarrow \frac{3x}{\sqrt{x^2+1}} - 2 = 0 \Rightarrow \frac{3x - 2\sqrt{x^2+1}}{\sqrt{x^2+1}} = 0 \Rightarrow$$

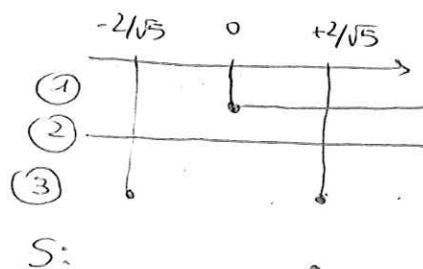
$$\Rightarrow 3x - 2\sqrt{x^2+1} = 0 \Rightarrow 2\sqrt{x^2+1} = 3x \Rightarrow \sqrt{x^2+1} = \frac{3}{2}x \Rightarrow \begin{cases} \frac{3}{2}x \geq 0 & (1) \\ x^2+1 \geq 0 & (2) \\ x^2+1 = \frac{9}{4}x^2 & (3) \end{cases}$$

$$(1) \frac{3}{2}x \geq 0 \Rightarrow x \geq 0$$

$$(2) x^2+1 \geq 0 \Rightarrow \forall x$$

$$(3) x^2+1 = \frac{9}{4}x^2 \Rightarrow \frac{9}{4}x^2 - x^2 - 1 = 0 \Rightarrow \frac{5}{4}x^2 - 1 = 0 \Rightarrow x^2 = \frac{4}{5} \Rightarrow x_{1/2} = \pm \frac{2}{\sqrt{5}}$$

○ grafico delle soluzioni del sistema



$\Rightarrow$ la unica soluzione è $x = \frac{2}{\sqrt{5}}$

$\Rightarrow A(\frac{2}{\sqrt{5}}; 0)$ int. assex

INT. ASSEX

$$\begin{cases} x=0 \\ y = \frac{3x}{\sqrt{x^2+1}} - 2 \end{cases} \Rightarrow y = \frac{3 \cdot 0}{\sqrt{0^2+1}} - 2 = 0 - 2 = -2 \quad B(0; -2) \text{ INT. ASSI}$$

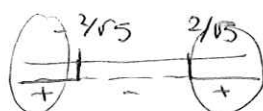
SEGNO

$$\frac{3x}{\sqrt{x^2+1}} - 2 \geq 0 \Rightarrow \frac{3x - 2\sqrt{x^2+1}}{\sqrt{x^2+1}} \geq 0 \begin{cases} D \sqrt{x^2+1} > 0 & \forall x \in \mathbb{R} \\ N 3x - 2\sqrt{x^2+1} \geq 0 \Rightarrow 3x \geq 2\sqrt{x^2+1} \Rightarrow \end{cases}$$

$$\Rightarrow \frac{3}{2}x \geq \sqrt{x^2+1}$$

$$\begin{cases} \frac{3}{2}x \geq 0 \Rightarrow x \geq 0 & (1) \\ x^2+1 \geq 0 & \forall x & (2) \\ x^2+1 \leq \frac{9}{4}x^2 \Rightarrow \frac{9}{4}x^2 - x^2 - 1 \geq 0 & \frac{5}{4}x^2 - 1 \geq 0 \end{cases}$$

$$\Rightarrow \frac{5}{4}x^2 - 1 = 0 \Rightarrow \frac{5}{4}x^2 = 1 \Rightarrow x^2 = \frac{4}{5} \Rightarrow x_{1/2} = \pm \frac{2}{\sqrt{5}}$$



$$\Rightarrow (3) x \leq -\frac{2}{\sqrt{5}} \quad x \geq \frac{2}{\sqrt{5}}$$

grafico di abitudine del sistema (*)

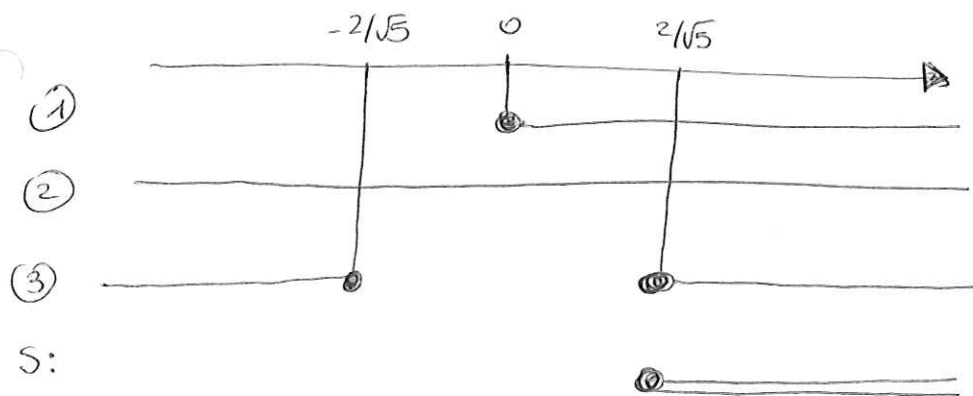
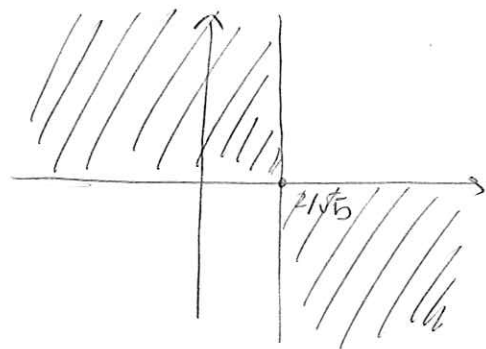
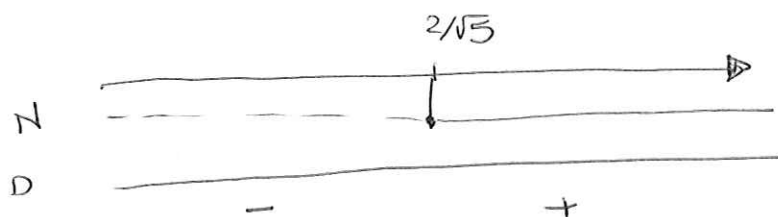


grafico del segno $\frac{N}{D}$



Limiti

$$\lim_{x \rightarrow -\infty} \frac{3x}{\sqrt{x^2+1}} - 2 = \lim_{x \rightarrow -\infty} \frac{3x}{x\sqrt{1+\frac{1}{x^2}}} - 2 = \lim_{x \rightarrow -\infty} \frac{3}{\sqrt{1+\frac{1}{x^2}}} - 2 = \frac{3}{1} - 2 = 1 \quad \text{A.O.}$$

Sanno
a $-\infty$
 $\Rightarrow |x| = -x$

$$\lim_{x \rightarrow +\infty} \frac{3x}{\sqrt{x^2+1}} - 2 = \lim_{x \rightarrow +\infty} \frac{3x}{x\sqrt{1+\frac{1}{x^2}}} - 2 = \frac{3}{1} - 2 = 1 \quad \text{A.O.}$$

Sanno a $+\infty$
 $\Rightarrow |x| = x$

DERIVATA PRIMA

$$y' = \frac{3 \cdot \sqrt{x^2+1} - 3x \cdot \frac{1}{\sqrt{x^2+1}}}{x^2+1} = \frac{3\sqrt{x^2+1} \cdot \sqrt{x^2+1} - 3x^2}{\sqrt{x^2+1} \cdot (x^2+1)} = \frac{3(x^2+1) - 3x^2}{(x^2+1)\sqrt{x^2+1}}$$

$$= \frac{3(x^2+1) - 3x^2}{(x^2+1)\sqrt{x^2+1}} = \frac{3x^2 + 3 - 3x^2}{(x^2+1)\sqrt{x^2+1}} = \frac{3}{(x^2+1)\sqrt{x^2+1}} \geq 0 \quad \forall x$$

DERIVATA SECONDA

$$y'' = \left(\frac{3}{(x^2+1)\sqrt{x^2+1}} \right)' = \left(\frac{3}{\sqrt{(x^2+1)^3}} \right)' = \left(\frac{3}{(x^2+1)^{3/2}} \right)'$$

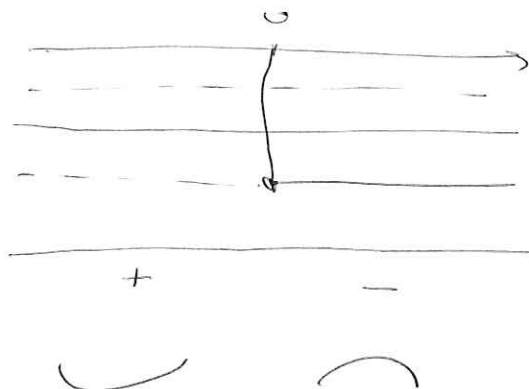
$$= \frac{-3 \cdot \frac{3}{2} (x^2+1)^{\frac{3}{2}-1} \cdot 2x}{(x^2+1)^3} = \frac{-9 (x^2+1)^{1/2} \cdot x}{(x^2+1)^3} = \frac{-9 \cdot \sqrt{x^2+1} \cdot x}{(x^2+1)^3} \geq 0$$

$$-9 \geq 0 \text{ mai}$$

$$\sqrt{x^2+1} \geq 0 \text{ sempre}$$

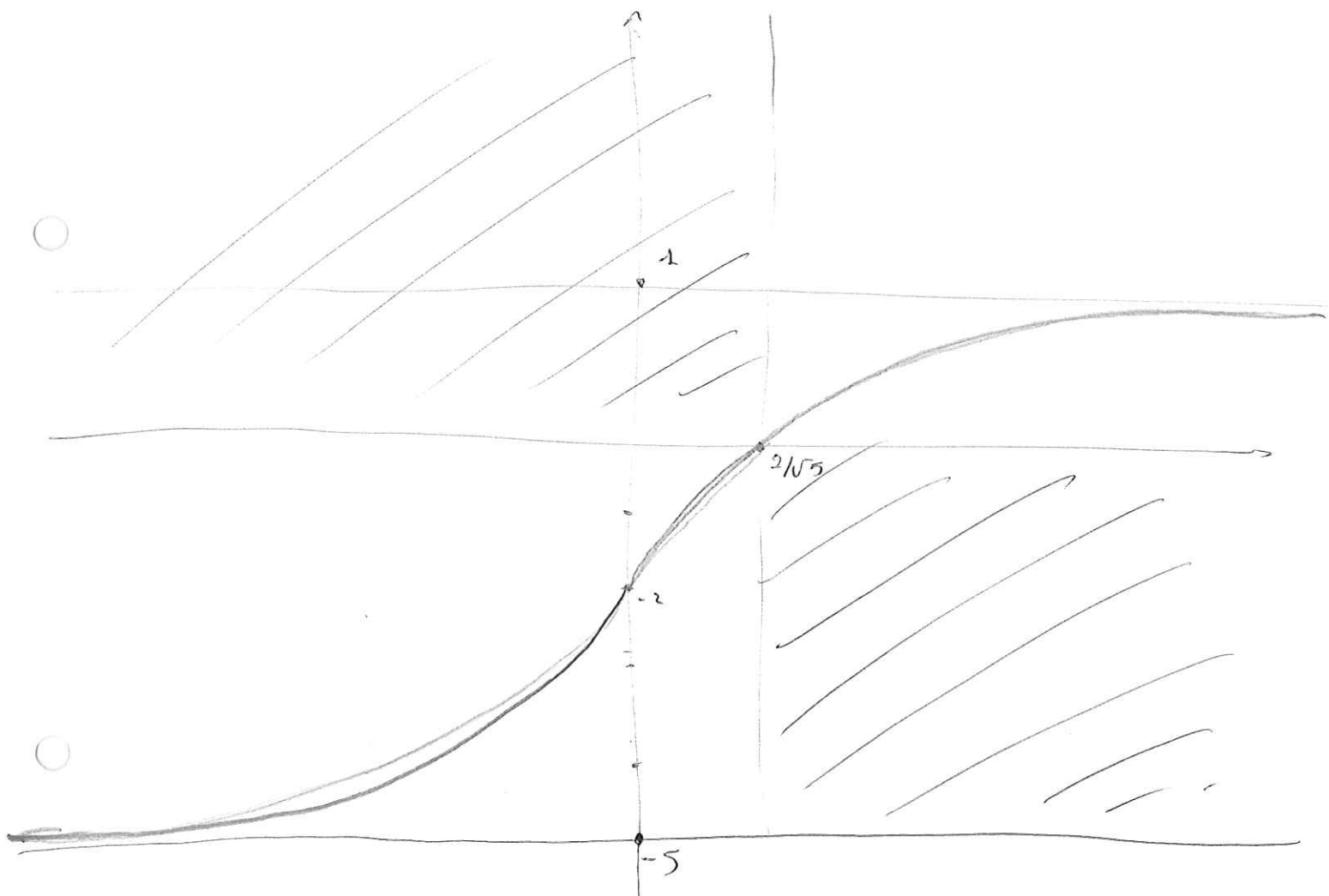
$$x \geq 0$$

$$(x^2+1)^3 \geq 0 \Rightarrow x^2+1 \geq 0 \text{ sempre}$$



$$x=0 \Rightarrow y=-2$$

$(0, -2)$ flesso



$$y = \frac{4}{3x - \sin x} + 2x$$

$$y' = \frac{-4(3 - \cos x)}{(3x - \sin x)^2} + 2$$

$$y'' = \frac{-4(+\sin x) \cdot (3x - \sin x)^2 + 4(3 - \cos x) \cdot 2(3x - \sin x)(3 - \cos x)}{(3x - \sin x)^4} =$$

$$= \frac{(3x - \sin x) \cdot [-4\sin x(3x - \sin x) + 8(3 - \cos x)^2]}{(3x - \sin x)^4} =$$

$$= \frac{-12x\sin x + 4\sin^2 x + 8(9 + \cos^2 x - 6\cos x)}{(3x - \sin x)^3} =$$

$$= \frac{-12x\sin x + 4\sin^2 x + 72 + 8\cos^2 x - 48\cos x}{(3x - \sin x)^3} =$$

$$= \frac{-12x\sin x + 72 - 48\cos x + \underbrace{4\sin^2 x + 4\cos^2 x}_{=4} + 4\cos^2 x}{(3x - \sin x)^3} =$$

$$= \frac{-12x\sin x + 72 - 48\cos x + 4 + 4\cos^2 x}{(3x - \sin x)^3} =$$

$$= \frac{-12x\sin x + 76 - 48\cos x + 4\cos^2 x}{(3x - \sin x)^3}$$

$$\lim_{x \rightarrow -\infty} \frac{3x-4}{\sqrt{16x^2+3x+9}} + 5 = \lim_{x \rightarrow -\infty} \frac{3x-4}{\sqrt{x^2(16-\frac{3}{x}+\frac{9}{x^2})}} + 5 =$$

$$= \lim_{x \rightarrow -\infty} \frac{3x-4}{|x| \sqrt{16 - \frac{3}{x} + \frac{9}{x^2}}} + 5 = \lim_{x \rightarrow -\infty} \frac{3x-4}{-x \sqrt{16}} + 5 =$$

$\downarrow \quad \downarrow$
 $0 \quad 0$

$$= \lim_{x \rightarrow -\infty} \frac{3x-4}{-4x} + 5 = -\frac{3}{4} + 5 = \frac{-3+20}{4} = \frac{17}{4}$$

$$\lim_{x \rightarrow +\infty} \frac{3x-4}{+4x} + 5 = \frac{3}{4} + 5 = \frac{3+20}{4} = \frac{23}{4}$$

dl. la $f(x)$ e definita in $]-\infty; +\infty[$

$\Rightarrow$ $\nexists$ as. verticali

$$\int_0^{\frac{\pi}{2}} \underbrace{e^{3x}}_{(1)} \underbrace{\cos 4x}_{(2)} + \underbrace{3x+4}_{(3)} dx$$

$$\textcircled{1} \int e^{3x} \cos 4x dx = \left\| \begin{array}{l} f'(x) = e^{3x} \Rightarrow f(x) = \frac{e^{3x}}{3} \\ g(x) = \cos 4x \Rightarrow g'(x) = -4 \sin 4x \end{array} \right\| =$$

$$= \frac{e^{3x}}{3} \cos 4x - \int \frac{e^{3x}}{3} (-4 \sin 4x) dx = \frac{e^{3x}}{3} \cos 4x + \frac{4}{3} \int e^{3x} \sin 4x dx =$$

$$= \left\| \begin{array}{l} f'(x) = e^{3x} \Rightarrow f(x) = \frac{e^{3x}}{3} \\ g(x) = \sin 4x \Rightarrow g'(x) = 4 \cos 4x \end{array} \right\| = \frac{e^{3x}}{3} \cos 4x + \frac{4}{3} \left(\frac{e^{3x}}{3} \sin 4x - \int \frac{e^{3x}}{3} \cdot 4 \cos 4x dx \right) =$$

$$= \frac{e^{3x}}{3} \cos 4x + \frac{4}{9} e^{3x} \sin 4x - \left(\frac{16}{9} \int e^{3x} \cos 4x dx \right)$$

$$\Rightarrow \int e^{3x} \cos 4x dx + \frac{16}{9} \int e^{3x} \cos 4x dx = \frac{e^{3x}}{3} \cos 4x + \frac{4}{9} e^{3x} \sin 4x \Rightarrow$$

$$\Rightarrow \left(1 + \frac{16}{9} \right) \int e^{3x} \cos 4x dx = \frac{e^{3x}}{3} \cos 4x + \frac{4}{9} e^{3x} \sin 4x \Rightarrow \left(1 + \frac{16}{9} = \frac{9+16}{9} = \frac{25}{9} \right)$$

$$\Rightarrow \int e^{3x} \cos 4x = \frac{9}{25} \left(\frac{e^{3x}}{3} \cos 4x + \frac{4}{9} e^{3x} \sin 4x \right) = \frac{3}{25} e^{3x} \cos 4x + \frac{4}{25} e^{3x} \sin 4x$$

VERIFICA

$$\left(\frac{3}{25} e^{3x} \cos 4x + \frac{4}{25} e^{3x} \sin 4x \right)' = \left(\frac{e^{3x}}{25} \cdot (3 \cos 4x + 4 \sin 4x) \right)' =$$

$$= \frac{1}{25} \cdot e^{3x} \cdot 3(3 \cos 4x + 4 \sin 4x) + \frac{e^{3x}}{25} (3 \cdot (-\sin 4x \cdot 4) + 4 \cdot (\cos 4x \cdot 4)) =$$

$$= \frac{e^{3x}}{25} [3(3 \cos 4x + 4 \sin 4x) + (-12 \sin 4x + 16 \cos 4x)] =$$

$$= \frac{e^{3x}}{25} [9 \cos 4x + 12 \sin 4x - 12 \sin 4x + 16 \cos 4x] =$$

$$= \frac{e^{3x}}{25} \cdot 25 \cos 4x = e^{3x} \cos 4x$$

VERIFICATO!

$$\textcircled{2} \int 3x \, dx = 3 \int x \, dx = \left(\frac{3x^2}{2} + C \right)$$

$$\textcircled{3} \int 4 \, dx = (4x + C)$$

quindi il risultato di $\int_0^{\pi/2} e^{3x} \cos 4x + 3x + 4 \, dx$ è : $\textcircled{1} + \textcircled{2} + \textcircled{3} =$

$$= \left[\frac{3}{25} e^{3x} \cos 4x + \frac{4}{25} e^{3x} \sin 4x + \frac{3}{2} x^2 + 4x \right]_0^{\pi/2} =$$

$$= \left(\frac{3}{25} e^{\left(\frac{3\pi}{2}\right)} \cos\left(4 \cdot \frac{\pi}{2}\right) + \frac{4}{25} e^{\left(\frac{3\pi}{2}\right)} \sin\left(4 \cdot \frac{\pi}{2}\right) + \frac{3}{2} \left(\frac{\pi}{2}\right)^2 + 4 \cdot \frac{\pi}{2} \right) -$$

$$- \left(\frac{3}{25} e^{3 \cdot 0} \cos(4 \cdot 0) + \frac{4}{25} e^{3 \cdot 0} \sin(4 \cdot 0) + \frac{3}{2} (0)^2 + 4 \cdot 0 \right) =$$

$$= \frac{3}{25} e^{\frac{3\pi}{2}} \cdot \cos(2\pi) + \frac{4}{25} e^{\frac{3\pi}{2}} \sin 2\pi + \frac{3}{2} \frac{\pi^2}{4} + 2\pi - \left(\frac{3}{25} e^0 \cos 0 + \frac{4}{25} e^0 \sin 0 + 0 + 0 \right) =$$

$$= \frac{3}{25} e^{\frac{3\pi}{2}} \cdot 1 + \frac{4}{25} e^{\frac{3\pi}{2}} \cdot 0 + \frac{3}{8} \pi^2 + 2\pi - \left(\frac{3}{25} \cdot 1 \cdot 1 + \frac{4}{25} \cdot 1 \cdot 0 \right) =$$

$$= \frac{3}{25} e^{\frac{3\pi}{2}} + \frac{3}{8} \pi^2 + 2\pi - \frac{3}{25}$$

$$13,35 + 3,70 + 6,28 - 0,12 = 23,21 \checkmark \text{ VERIFICATO}$$